SCALING OF PARTICLE PRODUCTION WITH NUMBER OF PARTICIPANTS IN HIGH-ENERGY A+A COLLISIONS IN THE PARTON-CASCADE MODEL

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In view of the recent WA98 data of π^0 spectra from central $Pb+Pb$ collisions at the CERN SPS, we analyze the production of neutral pions for $A+A$ collisions across the periodic table at $\sqrt{s} = 17$ A GeV and 200 A GeV within the framework of the parton-cascade model for relativistic heavy ion collisions. The multiplicity of the pions (having $p_\perp > 0.5$ GeV/c) in the central rapidity region, is seen to scale as $\sim (N_{part})^\alpha$, where N_{part} is the number of participating nucleons, which we have approximated as $2A$ for central collisions of identical nuclei. We argue that the deviation of α ($\simeq 1.2$) from unity may have its origin in the multiple scattering suffered by the partons. We also find that the constant of proportionality in the above scaling relation increases substantially in going from SPS to RHIC energies. This would imply that the (semi)hard partonic activity becomes a much cleaner signal above the soft particle production at the higher energy of RHIC, and thus much less dependent on the (lack of) understanding of the underlying soft physics background.

Recently, the WA98 Collaboration has published [1] data for the production of neutral pions up to transverse momenta of $p_\perp \simeq 4$ GeV/c, in central $Pb+Pb$ collisions at 160 A GeV/c incident momentum, corresponding to $\sqrt{s} \simeq 17$ A GeV. Two most interesting features of these data emerge when compared to corresponding data from pp collisions and collisions involving lighter nuclei [1]: a) an approximate invariance of the spectral shapes, i.e., a near indepence of the slope of the neutral pion $p_\perp$ spectra; b) a simple scaling of the π^0 with the number of participating nucleons, if the number of participants is large ($\gtrsim 30$).

Soon after its publication the (preliminary) WA98 data [2], was analyzed in two interesting papers, one by Wang [3] and the other by Gyulassy and Levai [4], with regard to the contribution to π^0 production at larger $p_\perp \gtrsim 2$ GeV/c due to (semi)hard parton scatterings and associated minijet production. Astonishingly, Wang was able to reproduce the WA98 data for the invariant π^0 cross-section by simply using the perturbative QCD cross-section for a single parton-parton scattering convoluted with the nuclear overlap function and a quark/hadron fragmentation function, and a parametrized accounting of the p_T-kick suffered by the partons. On the other hand, Gyulassy and Levai made a detailed numerical simulation using the event generator HIJING, and claimed that the consideration of multiple scattering effects of partons are crucial and not at all in conflict with the WA98 data.

In the present paper, we analyze the WA98 data from yet another angle [5]. We would like to demonstrate that the observed scaling of the pion yield in $A+A$ collisions as well as the insensitivity of the $p_\perp$ slope to a variation of A, *does not at all* imply the absence of dense matter effects on the particles, *nor* does it necessarily lead to the conclusion of Ref. [1] that the produced matter is thermally equilibrated. Gyulassy and Levai have already hinted this, but our findings go even further. In fact, a significant partonic cascade activity which scales roughly with $A^{4/3}$ (rather than with A) does *not* lead to a contradiction with the WA98 data, rather, we believe that it is confirmed by the WA98 results.

In order to elucidate this apparent mystery, we use the event generator VNI [6] which embodies the physics of the parton-cascade model [7] for ultra-relativistic heavy-ion collisions. The model attempts to describe the nuclear dynamics on the microscopic level of particle transport and interactions, by evolving the multi-particle system in space-time from the instant of nuclear overlap all the way to the final-state hadron yield. For details we refer the interested reader to Refs. [6]. Here it suffices to summarize the essential elements embodied in the model, namely: (i) the initial minijet production through liberation of partons from the colliding nuclei by means of (semi)hard parton scatterings; (ii) the subsequent parton cascading consisting of multiple gluon emission and successive rescatterings; (iii) the coalescence of final-state partons to color-neutral pre-hadronic clusters; (iv) the decay of clusters into primary hadrons; (v) the subsequent hadron cascading consisting of reinteractions of hadrons and clusters with resonance production and decay.

In this model, 'dilute' collision systems (involving beams of protons or light nuclei) naturally show a very different dynamical evolution than 'dense' collision systems (head-on collisions of heavy nuclei). Whereas in the former case multiple particle interactions are absent or negligible, in the case of heavy ions, the space-time development is characterized by multiple scatterings of partons, and later also of hadrons which are formed when the partonic system hadronizes. Moreover, the parton cascading during the early stage has rather distinct features, as compared to the hadron cascading during the late stage of the collisions. Multiple interactions among

produced hadrons mostly alter only the momentum distributions [8], and do not lead to a substantial increase of particle production. The interactions among partons, on the other hand, have a very different effect on particle production. Perturbative QCD tells us that primary (semi)hard scatterings of partons with momentum transfer $q_\perp \gtrsim 1-2$ GeV/c contribute the bulk of minijets. Moreover, the so kicked-out partons are excited off-shell by an amount $Q^2 \simeq q_\perp^2$, which they tend to shake off by gluon brems-strahlung. Thus, even in the absence of partonic rescatterings, the additional emission of gluons leads to an increase of the parton multiplicity on top of the number of those quarks and gluons which are liberated by the primary scatterings. In the parton-cascade model the characteristics of parton multiplication is further pronounced due to secondary interactions: Firstly, the initially kicked-out primary partons may rescatter and receive additional momentum transfer that again feeds the gluon emission. Secondly, all the newly produced off-spring of secondary partons increase the local density and can themselves re-interact. However, upon hadronization, the partonic color charges have to reorganize themselves to color-singlet composite objects that are the seeds of the emerging hadrons. Clearly, in order to make a hadron of mass m_h, the total invariant mass of the recombined partons must be at least equal to m_h, and consequently it requires in the mean more than just two partons to satisfy this kinematic condition, especially since the bulk of gluons from radiative emission piles up at the low-energy end of the perturbative regime. Nevertheless one would expect a proportionality between produced partons and resulting hadrons.

Indeed, the cluster-hadronization scheme [9], employed here to convert the outcome of the parton cascade into hadronic states, provides that the number of produced pre-hadronic clusters, and hence hadrons is proportional to the number of partons present in the system. Thus, the production of particles should reflect the extent of partonic scattering throughout the nuclear collision. This can be a very important consideration as the number of collisions among the primary partons has been shown to scale linearly with the number of the participating nucleons in the nuclear collision [10]. Thus a deviation of the number of produced hadrons from a similar linear scaling would be a direct confirmation of multiple scatterings taking place in the wake of relativistic collision of nuclei.

A simple consideration may illustrate these features of particle production. Let x denote the number of partons in each nucleon, and let each parton suffer ν collisions during the partonic stage. Assuming that each virtual parton radiates r partons, we see that the number of produced partons will vary as

$$N_{\rm partons} \;\propto\; \nu\,(1+r)\;x\;A\;, \qquad (1)$$

and if, as proclaimed,

$$N_{\rm hadrons} \;\propto\; N_{\rm partons}\;, \qquad (2)$$

one realizes immediately that if the partons interact only once, the multiplicity of the partons, and hence the multiplicity of hadrons, will scale as A. It is also clear that if every parton interacts with with every other parton then $\nu \propto A$, and the number of materialized partons would scale as A^2. That can happen, if the system would live for an infinitely long time. However, this is not the case. Rather than that, in relativistic heavy ion collisions, the partonic matter will expand, dilute, and eventually convert into hadrons. Thus a given parton may undergo $\nu \sim R/\lambda$ interactions; where R is the transverse size of the system and λ is the mean free path of the parton. Noting that $R \sim A^{1/3}$, we immediately see that the number of materialized partons, and hence the number of produced particles would scale as $\approx A^{4/3}$. An experimental verification of this scaling behaviour could be a direct manifestation the formation of a dense partonic matter!

We shall demonstrate now that these simple considerations are indeed confirmed by a detailed simulation with the event generator VNI on the basis of the parton-cascade/cluster-hadronization model. We first consider the recently measured transverse momentum distribution of π^0-production in central collsions of $Pb+Pb$ at CERN SPS obtained by the WA98 collaboration [1]. To make contact with the experimental data, the simulations were done for the range of impact parameters $0 < b < 4.5$ fm, which corresponds to 10% of minimum-bias cross-section. The result of our model calculation, shown as the solid histogram in Fig. 1a, is seen to be in decent agreement with the experimental measurements. The model results do not include the final-state interaction among produced hadrons yet, but it is likely [8] that the agreement will further improve once the effect of cascading hadrons is included. The dashed histogram in Fig. 1a corresponds to pp collisions at $\sqrt{s} = 17$ GeV, and scaled accordingly. Comparing $Pb+Pb$ with pp, clearly exhibits the well-known enhancement of the production of pions having large $p_\perp$ due to multiple scatterings among the partons [10].

As mentioned already, one of the interesting observations of the WA98 results is near independence of the slope of the $p_\perp$ spectra for the pions once the number of participants is large. In Fig. 1b we plot our results for the $p_\perp$ spectra of π^0's for a number of central AA collisions at $\sqrt{s} = 17$ A GeV for various $A+A$ systems from $A = 16$ to $A = 197$. One observes that they are almost identical in shape with a universal slope for $p_\perp \lesssim 1.5$ GeV/c. On the other hand, the deviations appearing at larger $p_\perp$ for heavier systems are indicative of enhanced multiple scattering there. Similar results (not displayed here) were obtained at RHIC energies.

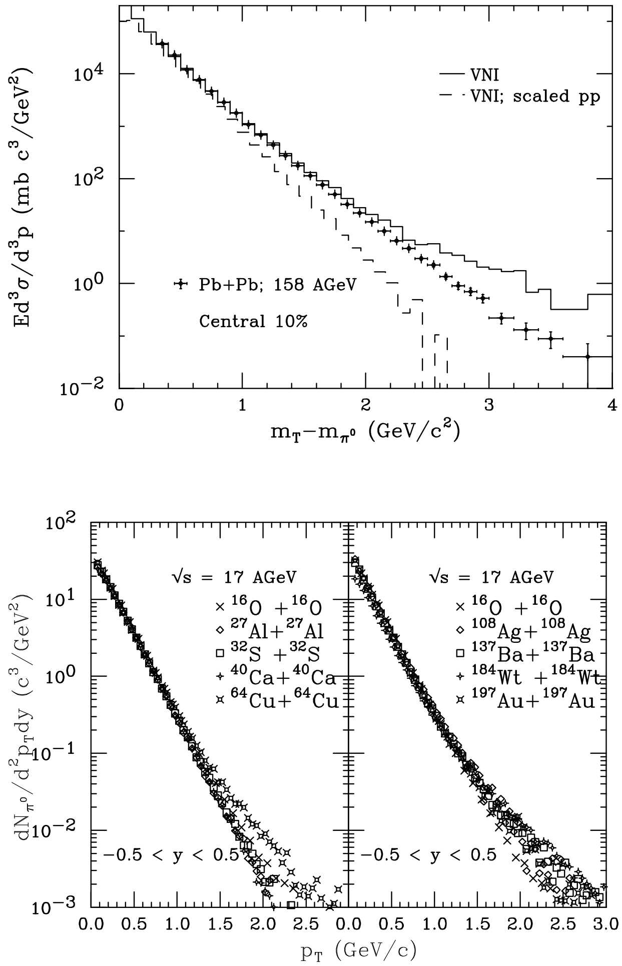

FIG. 1. *a) Top*: Transverse mass spectra of neutral pions in central (10% of minimum bias) collision of 158 AGeV $Pb + Pb$ collisions. The solid histogram represents our result from the parton-cascade/cluster-hadronization model. The dashed histogram corresponds to pp collsions at $E_{cm} = 17$ GeV, scaled accordingly. *b) Bottom*: Transverse momentum spectra of neutral pions in central collisions of identical nuclei at $E_{cm} = 17$ A GeV. The symbols are our results for various systems, from $O+O$ up to $Au+Au$. All results are normalized to the case $A = 16$.

In order to verify this scaling more closely, we have calculated, as a function of the nuclear mass number A, the production of π^0's in the central rapidity region ($-0.5 < y < 0.5$) having transverse momenta $p_\perp \geq 0.5$ GeV/c. The latter choice minimizes the influence of pions having their origin in decay of resonances. This kinematic window was motivated [1] by the WA98 collaboration in their measurement of the π^0 yield. Fig. 2a displays the simulation results for central $A + A$ collisions across the periodic table, at CERN SPS center-of-mass energy $\sqrt{s} = 17$ A GeV, while Fig. 2b shows the same for RHIC energy $\sqrt{s} = 200$ A GeV. The solid lines are fits to the model results, represented by the symbols, and scale as

$$N_{\pi^0} \propto (N_{part})^\alpha , \tag{3}$$

where $N_{part} = 2A$ is the number of participating nucleons, and α being extracted as:

$$\alpha \approx \begin{cases} 1.16 & \text{at } \sqrt{s} = 17 \text{ A GeV} \\ 1.23 & \text{at } \sqrt{s} = 200 \text{ A GeV} \end{cases} . \tag{4}$$

In view of the uncertainties, we may say that $\alpha \approx 1.2$ is a fair number that characterizes our model results. This is quite similar [11] to the observation of the WA98 experiment at $\sqrt{s} \simeq 17$ A GeV, namely [1] $\alpha \approx 1.3$, with the number of participating nucleons estimated from the impact parameter, in collisions involving lead nuclei.

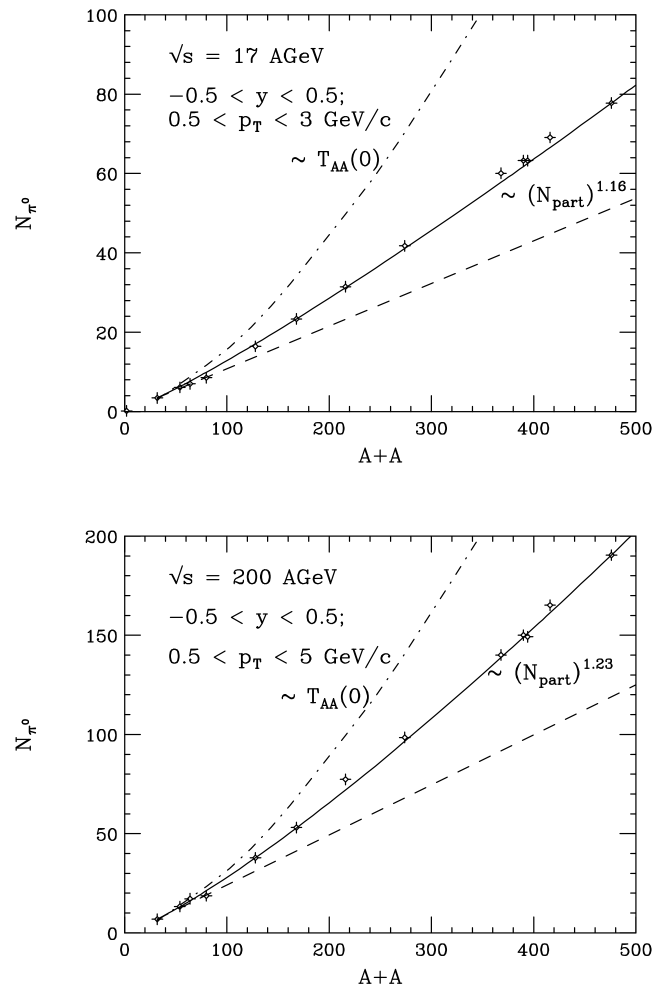

FIG. 2. *a) Top*: Mass number scaling of pion production in the central rapidity region at CERN SPS energies. The symbols represent the results of our simulations, and the solid curve is a fit to these results. *b) Bottom*: Mass number scaling of particle production in the central rapidity region at BNL RHIC energies. The symbols represent the results of our simulations, and the solid curve is a fit to these results. Dashed lines would correspond to a linear scaling, dotted-dashed lines to a scaling with the nuclear overlap $T_{AA}(b = 0)$.

For comparison, the dashed lines correspond to a linear scaling $\sim (N_{part})^{1.0}$, whereas the dashed-dotted lines indicate a hypothetical scaling with the nuclear overlap factor $T_{AA}(b = 0) \sim (N_{part})^{1.42}$. A linear scaling would reflect a single-collision situation, and a scaling with T_{AA} would indicate a Glauber-type multiple-collision scenario, in which nucleons suffer several collisions along their incident straight-line trajectory, without deflection but with energy loss. Comparing the three curves, we can conclude that our simulation results rise significantly slower than with T_{AA}, because firstly, the particles change direction through the collisions, and secondly, they are subject to a collision time of the order of the inverse momentum transfer, during which they cannot rescatter. On the other hand, our calculated π^0 yields grow much faster than linear with A, due to multiple scatterings. Hence, the scaling of the π^0 yield in the model may be interpreted as a *collision-meter* that indicates the extent of multiple interactions on the parton level (recall, that we did not include hadronic final-state interactions). Such an interpretation would also explain the (slightly) larger value of α, and also the constant of proportionality (cf. Fig. 2a and 2b) i.e. the stronger increase, at RHIC energy as compared to the results for CERN SPS energy. In the former case one would expect that multiple collisions are further enhanced due to a higher density of produced particles. This, we find, results in a increase larger than a factor of 2 in the number of π^0's between CERN SPS and RHIC.

In summary, we have demonstrated here that the observed scaling with A^α with $\alpha \simeq 1.3$ of the number of produced particles in heavy-ion $A + A$ collisions, as well as the approximate shape-independence of the transverse momentum spectra, are satisfactorily reproduced by the parton-cascade / cluster-hadronization model, though with a slightly smaller value of $\alpha \simeq 1.2$. The quark-gluon multiplicity from the early stage of the collisions due to initial minijet production *plus* subsequent cascading with multiple scatterings and gluon emissions, leads to a final-state particle multiplicity which approximately scales with the number of nucleon collisions (i.e., $\sim A^{4/3}$), as inferred by the WA98 experiment using high $p_\perp$ π^0's.

The *key result* of our model simulations is that multiple parton scatterings contribute significantly in collisions involving heavy nuclei. We have checked that the initial minijet production component alone, without subsequent cascading and reinteractions of the minijets, would yield for heavy ions a π^0 yield as for the *pp* case in Fig. 1a, which would clearly underestimate the WA98 $Pb + Pb$ data.

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[5] An incorrect π^0 yield was reported by us earlier in Phys. Rev. C**56**, 2718 (1997). A computational error in an earlier version of the simulation program VNI resulted in a substantial overestimation of π^0's, while all other hadron species were unaffected.

[6] K. Geiger, Comp. Phys. Com. **104**, 70 (1997); K. Geiger, R. Longacre, and D. K. Srivastava, ⟨ nucl-th/9806102 ⟩. The latest version of the computer program VNI can be obtained from *http://rhic.phys.columbia.edu/rhic/vni*, or from the authors.

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[11] The difference between $\alpha \simeq 1.3$ of the WA98 measurement, and the value $\alpha \simeq 1.2$ extracted from our calculations, may be attributed to the specific way particle collisions are simulated in VNI: for each collision a characteristic collision time $\tau_{coll} = const./\sqrt{q^2}$ is estimated, depending on the invariant squared momentum transfer q^2. The prefactor is not known, so we take $const. = 1$, but one may consider this as a true parameter to be adjusted. We also note that the geometry of central $A + A$ collisions is with $2A$ participating nucleons is some what different from the case when A nucleons from a Pb projectile-nucleus interact with A nucleons from a Pb target-nucleus for some impact parmeter.